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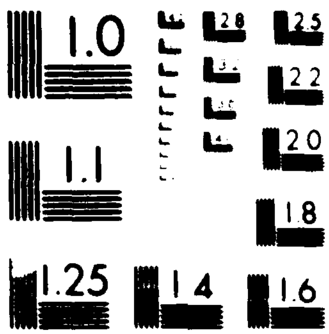

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STUDIES IN STATISTICAL SIGNAL PROCESSING

FINAL TECHNICAL REPORT

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This is the final report on Contract AFOSR-83-0228, which funded our research on statistical signal processing for a period of three years, between July 1, 1983 and June 30, 1986. Section II provides a description of work done in the period 1983-1985 and Section III describes recent results, obtained in 1985-1986.

I. Introduction

The primary objective of our research is to develop efficient and numerically stable algorithms for nonstationary signal processing problems by understanding and exploiting special structures, both deterministic and stochastic, in the problems. We also strive to establish and broaden links with related disciplines, such as cascade filter synthesis, scattering theory, numerical linear algebra, and mathematical operator theory for the purpose of cross fertilization of ideas and techniques. These explorations have led to new results both in estimation theory and in these other fields, e.g., to new orthogonal cascade digital filter structures, new algorithms for triangular and *QR* factorization of structured matrices and new techniques for stability testing.

For several years, the guiding principle in these studies has been the concept of (Toeplitz-oriented) displacement structure (Kailath, Kung and Morf, (1979)), which generalized and subsumed our earlier work on fast (Chandrasekhar) control and estimation algorithms for state-space models (Morf, Sidhu and Kailath, (1974)). Several authors have since picked up these ideas in a number of fields. A notable such work is a recent book by Heinig and K. Rost of East Germany, entitled "Algebraic Methods for Toeplitz-Like Matrices and Operators." A contribution of this book is the introduction of a number of different displacement rank concepts for not only Toeplitz-like operators but also Hankel-like, Vandermonde and Hilbert operators. Somewhat contemporaneously, in the Ph.D. research of H. Lev-Ari (parts of which were summarized in our AFOSR proposal submitted March 1983), we introduced a generating function characterization that also lends itself to generalization to a large class of structured matrices. Moreover, as we shall briefly describe below, this approach introduces a natural geometric significance to the theory, which allows a number of other interesting developments, e.g., studies of various problems in system theory, such as minimal realization, Padé approximation, control design, and a variety of root distribution (stability) problems for polynomials. Also, connections of the generating function approach to the theory of inverse scattering were clarified during this research, yielding useful generalizations and a unified framework for the derivation of algorithmic alternatives for the solution of the above-mentioned problems.

In particular, the topic of root distribution problems for polynomials has led our enquiries in a somewhat different direction. We started with some recent results of Y.

Bistritz (*Proc. IEEE*, September 1984), in which he obtains several new tests for the root distribution of polynomials with respect to the unit circle that needed only half the number of multiplications (and the same number of additions) as the well known Schur-Cohn (or equivalently the Jury-Marden table) test (Jury, (1964)). Now estimation theorists have long known that the Schur-Cohn test is essentially a reverse (degree-reducing) form of the fast Levinson algorithm for solving Toeplitz linear equations. It is therefore reasonable to expect that similar reductions in computational complexity could also be obtained for the Levinson algorithm itself, and this possibility has now been explored in some detail. In particular, we have developed three classes of reduced complexity Levinson-type algorithms and have demonstrated the relationship between them (Bistritz, Lev-Ari and Kailath, (1986)). Moreover, we have shown that these algorithms are essentially the only ones providing such complexity reduction. An interesting fact is that the saving is achieved by using three-term (rather than two-term) recursions and propagating them in an *Impedance/Admittance* domain rather than the conventional *scattering* domain. Our results apply both to Toeplitz and to close to Toeplitz systems. Moreover they provide a general method for reducing computational requirements in various recursive algorithms, e.g. adaptive least-square lattice algorithms. Bistritz is a postdoctoral scholar at Stanford, and these results and several related ones were developed with him.

The QR factorization of a matrix $\mathbf{A}$ is closely connected with the triangular factorization of $\mathbf{A}^*\mathbf{A}$. Recently we have shown that the displacement rank of a product of two matrices does not exceed the sum of the displacement ranks of the individual matrices. This result made it possible to develop a fast algorithm for QR

factorization of matrices with a displacement structure, and in particular for Toeplitz matrices (Chun, Kailath and Lev-Ari, (1986)).

II. Exploiting Generalized Displacement Structure

We originally started (in the late seventies) with the following definition. A symmetric matrix $\mathbf{R}$ has a *displacement structure* if the difference

$$\nabla\mathbf{R} := \mathbf{R} - \mathbf{ZRZ}^* , \quad \mathbf{Z} = [\delta_{i,j}]_{i,j=0}^{n}$$

has low rank. Note that $\mathbf{Z}$ is the lower shift matrix with ones on the first subdiagonal and zeros everywhere else. This definition was motivated by the fact that for both Toeplitz matrices and for *inverses* of Toeplitz matrices the *displacement rank* (i.e., the rank of $\nabla\mathbf{R}$) is 2. We have shown in previous work (largely supported by AFOSR) that the displacement concept was a key tool for developing fast algorithms of many kinds, including factorization and inversion of Toeplitz and near-Toeplitz matrices, and to fast (generalized Levinson and Schur) algorithms for solving linear systems with such coefficient matrices. Not surprisingly, these results led naturally to cascade orthogonal structures for the prediction of nonstationary processes (Lev-Ari and Kailath, (1984)). We have also found that the same concept is tightly connected to the more general problem of cascade filter synthesis in network theory and digital filtering as well as to a variety of inverse scattering problems (some references are Rao and Kailath, (1984, 1985), Bruckstein and Kailath (1986)).

Recently we have extended the concept of displacement structure to a very broad family of structured matrices, including Hankel matrices and their inverses, sums of Toeplitz and Hankel matrices and several others (Lev-Ari and Kailath, (1986)). The generalized displacement of a matrix $\mathbf{R}$, is defined as

$$\nabla_d\mathbf{R} := \sum_{k,l=0}^{N} d_{kl}\mathbf{Z}^k\mathbf{R}(\mathbf{Z}^*)^l \ . \tag{1a}$$

where the asterisk (*) denotes Hermitian transpose (complex conjugate for scalars). This is characterized by a (Hermitian) displacement matrix $\mathbf{J}_d$,

$$\mathbf{J}_d := \{d_{kl};\ 0 \le k,l \le N\} \tag{1b}$$

The previous notion of displacement corresponds to the particular displacement matrix

$$\mathbf{J}_d = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} := J_T$$

while the displacement notion used by Heinig and Rost (1984) for Hankel matrices, corresponds to the displacement matrix

$$\mathbf{J}_d = \begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix} := J_R$$

We have shown that efficient triangular factorization of a Hermitian matrix $\mathbf{R}$ can be formulated for all displacement matrices whose inertia is $(1,1)$. This means that $\mathbf{J}_d$ must have the form

$$\mathbf{J}_d = \Delta \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \Delta^* \tag{2}$$

where Δ is any matrix with two columns of arbitrary lengths.

The concept of displacement structure and its properties are more conveniently described in terms of *generating functions*. The generating function of a matrix $\mathbf{R}$ is a power series in two complex variables, viz.,

$$R(z,w) := [1 \;\; z \;\; z^2 \ldots]\mathbf{R}[1 \;\; w \;\; w^2 \ldots]^* \tag{3}$$

The displacement $\nabla_d \mathbf{R}$ of a matrix has the generating function $d(z,w)R(z,w)$, where $d(z,w)$ is the generating function of the Hermitian matrix $\mathbf{J}_d$, viz.,

$$d(z,w) = \sum_{k,l=0}^{N} d_{kl} z^k (w^*)^l \tag{4}$$

Thus the generating function of a Hermitian matrix with a displacement structure has the form

$$R(z,w) = \frac{G(z)\, J\, G^*(w)}{d(z,w)} \tag{5}$$

where J is any constant nonsingular Hermitian matrix. The triple $\{d(z,w), G(z), J\}$ is called a generator of $R(z,w)$.

We have shown (Lev-Ari and Kailath, (1986)) that efficient triangular factorization of $\mathbf{R}$ is possible if there exists a matrix function $\Theta(z)$ that satisfies the matrix equation

$$\Theta(z) J \Theta^*(w) = J - \frac{d(z,w)}{d(z,0)d(0,w)}\, JMJ \tag{6a}$$

where

$$M := G^*(0) R^{-1}(0,0) G(0) = M^* \tag{6b}$$

We have also shown that (6) has a solution if, and only if,

$$d(z,w) = \Delta(z) J_T \Delta^*(w) \tag{7}$$

which is the generating function version of (2). The (nonunique) solution of (6) is

$$\Theta(z) = \left\{ I - \frac{d(z,\tau)}{d(z,0)d(0,\tau)}\, JM \right\} U \tag{8a}$$

where U is any constant matrix such that

$$UJU^* = J \tag{8b}$$

and τ is any complex constant such that

$$d(\tau,\tau) = 0 \tag{8c}$$

The factorization of $\mathbf{R}$ is obtained via the recursion

$$G_{i+1}(z) = G_i(z)\Theta_i(z) \qquad i = 0,1,2,... \tag{9}$$

where $\Theta_i(z)$ have the form (8). This algorithm requires $O(n^2)$ computations to factor a structured $n \times n$ matrix, in contrast to the conventional LDL^* algorithm which requires $O(n^3)$ operations to factor an arbitrary matrix.

The generalization of the displacement concept has opened several new avenues of research, which we are currently pursuing. We have found that a family of so-called *Bezoutian matrices* arising in the theory of resultants of polynomials, with applications to topics such as testing for coprimeness, root distribution, compensator design, etc., also has a displacement structure. Moreover, this connection has suggested a new geometric interpretation for the displacement function $d(z,w)$. Since $d(z,z)$ is real for all z, we can define a partition of the complex plane into three mutually exclusive sets, viz.,

$$\begin{aligned} \Omega_+ &= \{z\ ;\ d(z,z) > 0\} \\ \Omega_0 &:= \{z\ ;\ d(z,z) = 0\} \\ \Omega_- &:= \{z\ ;\ d(z,z) < 0\} \end{aligned} \tag{10}$$

Notice that the curve Ω_0 is the boundary of the domains Ω_+, Ω_-. While the original displacement concept turns out to be associated with the unit circle T, and the Hankel displacement is associated with the real line R, other curves may also be considered. This observation has also revealed a surprising connection with the theory of orthogonal polynomials on arbitrary plane curves.

On the fundamental theoretical level we made connections with the work of L. De Branges, H. Dym and P. Dewilde on reproducing kernel Hilbert spaces associated with plane curves. The link connecting this topic with the work of Soviet mathematicians (Livsic, Brodskii, Krein, Potapov) on operator colligations and J-unitary operators appears to be the concept of systems (operators) that are J-lossless with request to a curve. This concept underlies our work on factorization and cascade filter synthesis, as well as the very recent work of Prabhakara-Rao and Dewilde (1984) on state space models for such systems. In fact, our equation (6) implies that $\Theta(z)$ is J-lossless, i.e.,

$$\begin{aligned} &\Theta(z)J\Theta^{*}(z) \leq J && z \in \Omega_{+} \\ &\Theta(z)J\Theta^{*}(z) = J && z \in \Omega_{0} \\ &\Theta(z)J\Theta^{*}(z) \geq J && z \in \Omega_{-} \end{aligned} \tag{11}$$

It is interesting to notice that Prabhakara-Rao and Dewilde start with (11) and show that any finite-state solution of (11) must essentially have the form (8).

III Recent Results

Immitance-Domain Three-Term Recursions

The structured generating function $R(z,w)$ is not affected if we replace the generator $\{d(z,w), G(z), J\}$ with the equivalent generator $\{d(z,w), G(z)T, T^{-1}JT^{-*}\}$ where T is an arbitrary nonsingular constant matrix. We may therefore choose T to minimize the computational requirements of our fast factorization procedure. In particular, we have observed that when $d(z,w) = 1 - zw^*$ (i.e., when Ω is the unit circle), then the choice

$$T = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \tag{12}$$

results in a 50% reduction in the number of multiplications involved in the procedure. We used this observation to transform the Levinson algorithm for Toeplitz and near Toeplitz matrices, viz.,

$$\begin{bmatrix} a_n(z) \\ b_n(z) \end{bmatrix} = \begin{bmatrix} 1 & -k_n \\ -k_n & 1 \end{bmatrix} \begin{bmatrix} z & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a_{n-1}(z) \\ b_{n-1}(z) \end{bmatrix} , \tag{13}$$

with initial conditions,

$$a_0(z) = 1 \quad , \quad b_0(z) = \rho \quad , \tag{14}$$

into a computationally-improved recursion, viz.,

$$\begin{bmatrix} f_n(z) \\ g_n(z) \end{bmatrix} = \frac{\psi_n}{2\psi_{n-1}} \begin{bmatrix} 1-k_n & 0 \\ 0 & 1+k_n \end{bmatrix} \begin{bmatrix} z+1 & z-1 \\ z-1 & z+1 \end{bmatrix} \begin{bmatrix} f_{n-1}(z) \\ g_{n-1}(z) \end{bmatrix} ,$$

$$f_0(z) = \psi_0(1+\rho) \quad , \quad g_0(z) = \psi_0(1-\rho) \quad . \tag{15}$$

involving the polynomials $\{f_n(z), g_n(z)\}$ which are obtained from $\{a_n(z), b_n(z)\}$ via the transformation

$$\begin{bmatrix} f_n(z) \\ g_n(z) \end{bmatrix} := \psi_n \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} a_n(z) \\ b_n(z) \end{bmatrix} , \quad \psi_n \neq 0 \tag{16}$$

where ψ_n is a suitable scaling factor.

A comparison of (14) with (15) shows that while the number of multiplications has been reduced by the transformation (13), the number of additions has, in fact, increased. It turns out that the increase in the number of additions can be avoided by converting the *two term* recursion (15), which involves both $f_n(z)$ and $g_n(z)$ into a *three term* recursion involving only one of these the polynomials. The three-term recursion for $f_n(z)$ has the form

$$f_{n+1}(z) = \frac{\psi_{n+1}(1-k_{n+1})}{\psi_n(1-k_n)} \cdot \left\{ (z+1) f_n(z) - \frac{\psi_n}{\psi_{n-1}} (1-k_n^2) z f_{n-1}(z) \right\} , \tag{17}$$

with a similar recursion for $g_n(z)$.

A suitable choice of the scaling factors ψ_n leaves only one nontrivial coefficient in this recursion. Since this can be done only in three ways, there are three computationally-efficient forms of the recursion (17), namely,

$$f_{n+1}(z) = \delta_n (z+1) f_n(z) - z f_{n-1}(z) ,$$

$$f_{n+1}(z) = (z+1) f_n(z) - \lambda_n z f_{n-1}(z) , \tag{18}$$

$$\lambda_{n+1} f_{n+1}(z) = (z+1) f_n(z) - z f_{n-1}(z) .$$

The conventional formulation of the Levinson recursion (14) can be related to transmission-line models (see, e.g., Kailath and Lev-Ari, (1984) and Bruckstein and

Kailath (SIAM Review, accepted, 1986/7)). In particular, the ratio $b_n(z)/a_n(z)$ can be interpreted as the *scattering function* of a transmission line consisting of a cascade of (uniform) sections with different characteristic impedances. On the other hand the ratio $g_n(z)/f_n(z)$ can be interpreted as the *impedance* (or admittance) *function* of the same transmission line. For this reason we shall say that the original recursion (14) is expressed in the *scattering domain*, whereas the transformed recursions (15) or (17) are expressed in the *immitance domain*.† We first derived immitance domain algorithms as inverse scattering procedures for discrete transmission-line models in Bruckstein and Kailath, SIAM Review, 1986/7, however we were not interested there in computational complexity issues and did not recognize the reduction in operation counts.

The immitance-domain version of the Levinson algorithm is also useful in the efficient solution of problems involving the so-called *singular predictor polynomials* (Delsarte, Genin, Kamp and Van Dooren, (1982)), such as Pisarenko's harmonic retrieval technique.

Bezoutians

Bezoutians are structured matrices **B** whose generating function has the form

$$B(z,w) = \frac{p(z)[q^{\#}(w)]^* - q(z)[p^{\#}(w)]^*}{[1 \quad z]J_d[1 \quad w]^*} \tag{19}$$

where $In\{J_d\} = \{1,1\}$, $p(z),q(z)$ are arbitrary complex polynomials, and the sharp (#) denotes a suitably defined polynomial transformation that reflects the zeros of a polynomial with respect to the *circle* Ω_0 defined by $d(z,w) = [1 \quad z]J_d[1 \quad w]^*$. In

†Bode (1945) coined the term *immitance* to denote both impedance and admittance

work in progress, we have shown that the square matrix **B** whose size equals $\max\{deg\ p, deg\ q\}$ has a rank deficiency equal to the degree of the greatest common divisor (divisor) of $p(z)q(z)$, and that our factorization procedure efficiently computes this gcd. Moreover, since J_d can be selected in many ways, we have a large variety of procedures at our disposal, with different computational requirements and numerical behavior. In addition, we can apply equivalence transformations of the form $[p(z) \quad q(z)] \rightarrow [p(z) \quad q(z)]T^{-1}$ to further modify our factorization procedure. For instance, we can use the transformation

$$T = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ j & -j \end{bmatrix}$$

to obtain the expression

$$B(z,w) = j\, \frac{b(z)[a^{\#}(w)]^{*} - a(z)[b^{\#}(w)]^{*}}{[1 \quad z]J_d[w \quad w]^{*}} \tag{20}$$

where $a(z), b(z)$ are arbitrary complex polynomials. Finally, we can transform the two-term factorization recursions into a three-term form and examine the complexity and numerical robustness of the resulting procedures.

When we make the specific choice $q(z) = p^{\#}(z)$ in (19) the Bezoutian becomes *Hermitian.* In that case, it is known that the inertia of **B** serves to locate the zeros of the complex polynomials $p(z)$ with respect to the curve Ω_0. We have a new simple proof of that result; moreover, our previously mentioned fast factorization procedures, both in two-term and in three-term form, can efficiently establish the inertia of **B**. We have gained some insight into the problem of singularity (i.e., singular leading minors in **B**) and shown how to avoid it in three-term recursions. We intend to apply this concept to study the occurrence of the singularities in two-term recursions and to devise a simple cure to this problem.

Symmetric polynomials play a central role in the theory of Bezoutians. These are polynomials for which $p^{\#}(z) = p(z)$. If we restrict $a(z)$ and $b(z)$ in (20) to be symmetric the Bezoutian becomes Hermitian, and consequently the computational requirements of its factorization procedure reduce by a factor of 2. Thus the gcd of two symmetric polynomials can be computed with half the number of computations required for arbitrary complex polynomials. In particular, if we choose $J_d = J_R$ in (20), the polynomials $a(z), b(z)$ must have *real* coefficients and the corresponding factorization procedure coincides with the partial realization algorithm of Kalman. We intend to exploit this observation to examine in detail the possibility of constructing computationally-improved alternatives to the known partial realization algorithm.

Another fascinating observation relates Bezoutians to measures defined in circles. We have shown that the inverse of a positive definite Bezoutian is the moment matrix of a positive measure defined on the same circle as the Bezoutian. This observation provides a key to the extension of the notion of Bezoutians to curves other than circles, which we intend to study in the future.

QR Factorization

The factorization of a matrix into a product **QR** where **Q** is an orthogonal matrix and **R** is upper triangular is a key step in the eigenanalysis of this matrix. It is also instrumental in the solution of linear systems of equations involving matrices that are not strongly regular. Recently Cybenko (1985) has proposed a method for computing the singular value decomposition of a matrix via QR factorization.

QR factorization of a matrix $\mathbf{A}$ is closely connected with the triangular factorization of $\mathbf{A}^*\mathbf{A}$ since

$$\mathbf{A}^*\mathbf{A} = \mathbf{R}^*\mathbf{Q}^*\mathbf{Q}\mathbf{R} = \mathbf{R}^*\mathbf{R}$$

which proves that $\mathbf{R}^*$ is the (unique) lower-triangular factor of $\mathbf{A}^*\mathbf{A}$. Thus QR factorization can be efficiently carried out if $\mathbf{A}^*\mathbf{A}$ has a displacement structure. We have shown, in fact, that if

$$\mathrm{rank}(\mathbf{A} - \mathbf{Z}\mathbf{A}\mathbf{Z}^*) = \alpha$$

then

$$\mathrm{rank}(\mathbf{A}^*\mathbf{A} - \mathbf{Z}\mathbf{A}^*\mathbf{A}\mathbf{Z}^*) \le 2\alpha + 1$$

so that the displacement structure for $\mathbf{A}$ is inherited by the product $\mathbf{A}^*\mathbf{A}$ (Chun, Kailath and Lev-Ari, (1986)). Moreover, we have shown how to construct a generator of $\mathbf{A}^*\mathbf{A}$ when a generator for $\mathbf{A}$ is known. Once as generator of $\mathbf{A}^*\mathbf{A}$ has been computed the QR factorization of $\mathbf{A}$ is obtained in $O(\alpha n^2)$ operations where α is the displacement rank of $\mathbf{A}$.

Our previous work in this area has focused on the Toplitz-oriented notion of displacement, i.e., on the displacement function $d(z,w) = 1 - zw^*$. This enabled us to derive procedures for fast QR factorization of Toeplitz and close to Toeplitz matrices. We intend to extend the same ideas to other displacement functions in order to broaden the scope of applicability of our fast QR factorization procedures, e.g., to Hankel and Vandermonde matrices.

In another direction we intend to examine the applicability of equivalence transformations and three-term recursions to further reduce the computational complexity of our QR factorization procedure.

Connection to Inverse Scattering Theory

Connections between scattering theory and the displacement structure of covariance matrices was a topic that received much attention during the past few years. It was discovered that there are natural mappings between transmission-line structures, and other wave-propagation models, and the structure of fast algorithms for factoring or inverting covariance matrices with displacement structures. These results are described in papers by Kailath, Bruckstein and Morgan (1986) on fast factorization via transmission-line models and Kailath and Lev-Ari (1985) on mappings between covariances and physical systems. It turned out as a result of this research that the factorization algorithms are in fact solving inverse scattering problems, problems that require the recovery of layered, one-dimensional, scattering medium properties from its response to a probing input. The input-response pairs are the scattering data from which the parameters of the layered medium have to be determined. It turns out that the most straightforward approach to inverse scattering problems is based on a careful analysis of the causal wave-propagation, combined with the local properties of the scattering medium. This approach led to the development of a unified theory of inverse scattering, based on difference, or, in the continuous case, on differential equations. Several interesting results in inverse scattering theory are discussed in the papers by Bruckstein, Levy and Kailath (1985), Bruckstein and Kailath (accepted to SIAM Review, 1986/7). The results encompass several differential algorithms that were discovered by various researchers working in different fields such as, geophysics, distributed systems and transmission-line synthesis, speech research connected to pressure-wave propagation in acoustic tubes, mathematics and mathematical physics.

The classical theory of inverse scattering, which starts from the physical problem of determining potentials from quantum scattering experiments, was however based on solving nested sets of integral equations (associated to the names of Gelfand-Levitan, Marchenko, Krein and Gopinath-Sondhi) rather than propagating differential algorithms. The connections between these approaches also became clear during this research, and led to the realization that when the structure of the matrix/integral equations is exploited i.e., when the so-called fast algorithms for solving matrix/integral equations having Toepliz, Hankel or Toeplitz+Hankel structures are used, we obtain difference/differential methods that closely resemble the direct differential methods that exploit the structure of the scattering medium directly. The algorithms are similar, however, not identical, and this was an important point to see. It was then observed that the fast algorithms derived in conjunction with integral equations-based methods for inverse scattering also exploit implicitly the medium structure, however in a different way: while the differential methods use "layer-peeling", i.e. they identify the next layer of the medium and then propagate the signals through it to synthesize scattering data for the medium portion starting one layer deeper, the fast algorithms for integral equations use the same scattering data and propagate it through the entire portion of medium that is already recovered. So, the information for identifying the next medium layer is gotten by propagating the original scattering data through the already identified portion of the scattering medium and when the next medium layer is identified it is adjoined to the already known medium section. For this reason we call the fast integral equations-based algorithms "layer adjoining" algorithms. The above-described alternatives for doing inverse scattering, i.e. layer-peeling and layer-adjoining turned out to be quite general processes that can be applied to a variety of problems. In fact we recognized that the classical problem of partial realization theory falls very nicely in this scattering framework, and the above inversion alternatives readily yield a unified picture of the algorithmic alternatives one has in solving this problem (see Bruckstein and Kailath, accepted ASSP Magazine paper, 1986/7).

Computationally, the layer-peeling and adjoining algorithms have the same complexity counts ($O(N^2)$); however the layer adjoining methods require the computation of inner-products (to propagate the original scattering data through the already identified layers). An inner product is a computational bottleneck in parallel implementations since a long addition requires $O(logN)$ time, even with N processors. The layer-peeling algorithms, which avoid explicit computations of inner products by propagating the scattering data through each identified layer, are more suitable for parallel implementations.

Direction Finding, Signal Resolution and Covariance Structures

Another avenue of research during the period of our contract was the use of covariance structures in processing signals received by an array of detectors. The basic results in this direction concentrated first on applying an algorithm of R. Semidt, called Multiple Signal Characterization or MUSIC, developed in the context of direction finding with antenna arrays, to spatio-temporal spectral analysis (Wax, Shan and Kailath, 1984) and to a wealth of signal resolution problems (Bruckstein, Shan and Kailath, 1985, and the thesis of M. Wax). Several important results were also obtained on the basic direction finding problem (the paper Optimum Localization of Multiple Sources in Passive Arrays, by Wax and Kailath, was awarded the senior paper award of the ASSP Society of the IEEE), on dealing with coherent sources, often arising in multipath situations, see. e.g. Shan, Wax and Kailath (1985), and on the determination of number of sources by information theoretic criteria (Wax and Kailath, 1985).

AFOSR SUPPORTED PUBLICATIONS - 1983 - 1986

Published Journal Papers

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